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Downward Wage Rigidity in a Model of Equal-Treatment Contracting

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Abstract

Following insights by Bewley (1999a), this paper analyses a model with downward rigidities in which firms cannot pay discriminate based on year of entry to a firm, and develops an equilibrium model of wages and unemployment. We solve for the dynamics of wages and unemployment under conditions of downward wage rigidity, where forward looking firms take into account these constraints. We show that there is a frontloading incentive which leads to a simple solution in the case of no uncertainty. Using productivity data from the post-war US economy, we analyse the ability of the model to match certain stylised labour market facts.

JEL Codes: E32, J41

Keywords: Labour contracts, business cycle, unemployment, equal treatment, downward rigidity, cross-contract restrictions.

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1 Introduction

Truman Bewley has argued that there are two key features constraining wage cuts for new hires in recessions (Bewley (1999a)). Because wage cuts for incumbents will have a negative impact on morale, firms avoid them under all but extreme circumstance; at the same time while new hires may be willing to work at a lower wage than that paid to incumbents, paying them less would disrupt internal equity and so their wages will be set at the same level as incumbents’:

New employees, in contrast, feel it is inequitable to be paid according to a scale lower than the one that applied to colleagues that were hired earlier. For this reason, downward pay rigidity for new hires exists only because the pay of existing employees is rigid. (Bewley (1999b))

Bewley’s account mainly concentrates on the question of why firms do not cut wages in recession. But it raises the important question of how forward looking firms take into account the fact that such constraints—downward rigidity combined with “equal treatment” of new hires—may arise in the future. For example, a firm, anticipating downward wage rigidity, may temper wage increases in better times. Indeed, one might ask in Bewley’s analysis what determines the initial wage which is too high once recession strikes. Or in more generality, and supposing that firms can offer long-term contracts, the firm must take into account these equal treatment constraints which may prevent it bringing in new hires at a low wage in downturns, and also prevent the firm hiring at a higher wage than that offered to incumbents when the labour market is tight.¹ What are the implications for wages and employment in an equilibrium model with forward looking firms and workers?

¹Bringing in workers at *higher* pay than incumbents is even more problematic; thus while—in contrast to the primary sector—Bewley found evidence that new hires are sometimes paid a lower rate than incumbents in the secondary sector, even there, paying new hires *more* than incumbents is deemed to be very disruptive (Bewley (1999a, p. 320)).

In this paper we attempt to accomplish two things. First we solve a dynamic equilibrium model in which firms take into account downward rigidity constraints. Secondly, we investigate whether the resulting wage and unemployment dynamics have reasonable properties when judged against US post-war experience.

In more detail: To analyse the dynamic consequences of Bewley’s insights, we analyse a model in which the pay of new hires and existing workers is linked within each firm—indeed is identical, given that we assume all workers are perfect substitutes—and in which the pay of incumbents is subject to some downward rigidity. This rigidity is then transmitted to the pay of new hires.² Workers and firms then must anticipate the effects of this, so that for example with full downward rigidity, an increase in current wages means that future wages cannot be cut below this level. Despite the enormous literature which exists on downward rigidities, there has been almost no analysis of the forward looking nature of the decision problem and its labour market implications (the only exception we are aware of is Elsby (2009), who solves a problem involving downward nominal rigidities, but in a very different context).

To do this, we take the equal treatment model of Snell and Thomas (2009) and add an explicit downward rigidity constraint.³ We do not, it should be stressed, attempt to provide an explicit foundation for the two constraints.⁴ In the unrestricted model—without the downward rigidity restriction—because of equal treatment of workers firms have to trade-off the desire to insure their risk-averse workers against

²We stress the point that for there to be significant labour market implications it is necessary that downward rigidity applies to new hires—if it only applies to incumbent workers so that new hires can be hired at a flexible wage, there is no reason why hiring decisions should not be efficient.

³We also extend that model to multiple sectors, an extension which is crucial for addressing recent empirical evidence.

⁴Snell and Thomas (2009) show how equal treatment can be endogenised if the contracting environment is “at will” and a similar argument could be used here. A closely related approach is developed by Menzio and Moen (2008). Similar issues arise in the insider-outsider literature; see Gottfries and Sjöström (2000).

the need to respond to market conditions to not only prevent their workers from quitting, but also to take advantage of states of the world where labour is cheap. The insurance motive alone provides a degree of downward rigidity. In this paper we add to this model downward wage restrictions in the spirit of Bewley to see how the performance of the model changes. A firm will face not only a sequence of participation constraints if it needs to retain existing, or hire in new, workers, but also a constraint each period restricting the degree to which it can cut its wage. A major challenge of the analysis is to solve for an equilibrium in the face of these constraints.

We show that the combination of equal treatment and sufficient labour turnover leads to a frontloading incentive. If a firm is not constrained next period, either by the participation constraint or by the downward rigidity constraint, then it will want to increase current wages at the expense of future wages. The reason for this is that a worker employed today puts less weight on future wages than does the firm: because of (exogenous) turnover there is a chance that the worker will not receive the future wage, but the firm will still pay—by equal treatment—the same wage to a new hire who replaces this worker. For the case where there is no uncertainty (so firms perfectly anticipate all future productivity changes) this allows us to deduce that the wage will always be up against one of the constraints and we are able to provide a complete characterization of equilibrium provided that the downward rigidity constraint is not too loose (does not allow too large a reduction in real wages). Unfortunately this result doesn't fully generalise to the case with uncertainty, although we present a result for a two-period model.

The equal treatment assumption prevents firms from cutting wages for new entrants, so that in periods with adverse shocks the wage may not fall sufficiently to clear the labour market. In the absence of equal treatment, wage rigidity in this model (with fixed hours) would not have employment effects, as utilities offered to

new entrants would be flexible. We show however that (under certain conditions) firms hire up to the point where the real wage equals the marginal product of labour; to the extent then that wages do not correspond to market-clearing levels hiring will be inefficient; in fact we show that this occurs only in the direction of wages being too high leading to inefficiently low employment and an excess supply of labour.

Not only do we solve for an equilibrium of a model when forward looking firms have downward constraints, but we also look at whether the equilibrium wage and unemployment dynamics look reasonable. We argue that our model, when simulated with sectoral productivity shocks, is capable of generating simulation results that are reasonably consistent with empirical unemployment and wage movements over the business cycle. We find that this model produces more reasonable wage-unemployment dynamics than Snell and Thomas (2009).

An outline of the paper is as follows. We start by looking at whether there is evidence for the equal treatment assumption in Section 2. Then we lay out the basic assumptions of our model in Section 3 and the next two subsections. In Section 3.1.2 we prove a simple characterization of the equilibrium when there is no uncertainty. We discuss the basic results in Section 3.2. In Section 3.2.1 we discuss to what extent these results extend to the uncertainty case. In Section 4 we simulate the model using sectoral TFP data from the postwar US economy to generate predictions of wage and unemployment movements, and see to what extent these satisfy certain stylised facts. Finally Section 5 contains concluding comments.

2 Evidence for equal treatment

While we simply impose that all workers within a firm must be paid the same, we here briefly review some empirical evidence on the issue of equal treatment. An early study was Baker, Gibbs, and Holmstrom (1994), who examined the pay

of managerial employees in a single firm over time. They found that incumbents' pay tends to move together, but the pay of entrants is significantly more variable, suggesting that the pay of new hires may be more subject to outside conditions than that of incumbents. However they do not formally control for composition of entry cohorts, so it is difficult to know what the cause of this extra variability is. Wachter and Bender (2008) have recently run a similar analysis on a number of firms in a large German manufacturing sector, and they, too, find evidence of substantial and quite persistent entry cohort effects. However these seem to be widely distributed across firms at any given date, suggesting, as they note, that they are not driven by cyclical phenomena. A study by Kwon and Milgrom (2005) of Swedish workers finds that if cohort effects for labour market entry and occupation entry are included in addition to firm entry cohort effects, the former two are procyclical in line with expectation, while the latter actually appear *countercyclical*. In other words, a worker entering the labour market in a downturn will tend to do worse than those already active, but entering a firm in a downturn does not of itself lead to a lower wage than that received by incumbents; in fact the opposite appears to be the case. Haefke, Sonntag, and van Rens (2007) argue that wages of those entering the labour market from non-employment are considerably more variable than those who remain in employment. This does not, though, imply that new hires are treated differently within particular firms. In a similar vein, Pissarides (2007) summarises empirical studies that find wages for workers who change jobs are considerably more procyclical than those who remain with the same employer. Gertler and Trigari (2009), however, argue that such studies are not demonstrating that the new hire wage is more procyclical than a stayer's wage *within a particular firm*. The reason for the empirical finding could simply be that match quality varies procyclically.

Finally, as discussed above, survey evidence in Bewley (1999a) suggests that violations of equal treatment are unusual, particularly in the primary sector. Similar

findings exist for other countries: “Managers responded that hiring underbidders would violate their internal wage policy” (Agell and Lundborg (2003, p.7), based on a Swedish survey); in a British survey, Kaufman (1984) reported that almost all managers viewed bringing in similarly qualified workers at lower wage rates as “infeasible.” Akerlof and Yellen (1990) argue that personnel management texts treat the need for equitable pay as virtually self-evident.

3 The model

The model is as follows. Time runs from $t = 1, 2, 3, \dots, T$, where $T \geq 2$ is finite, and there is a single consumption good each period (all wages below are real wages). There are M equal sized sectors. All workers are assumed to be identical, except for the date at which they enter the labour market, and the sector to which they are currently associated (we abstract from any tenure or experience effects on productivity). Workers are risk averse with per period twice differentiable utility function $u(w)$, $u' > 0$, $u'' < 0$, where $w \geq 0$ is the income which must be consumed within the period; it is assumed that they can neither save nor borrow. There is no disutility of work, but hours are fixed so that workers are either employed or unemployed. Assume that if workers are not employed in a period, they receive some low consumption level $\underline{c} > 0$. There is a large (but fixed) number of identical risk-neutral firms in each sector. A firm in sector m has a diminishing returns technology where output is $F^m(N, s_t)$ with $\partial F^m / \partial N > 0$, $\partial^2 F^m / \partial N^2 < 0$, where N is labour input and s_t is the current shock (which specifies current productivity in each sector). It is assumed that a firm must always employ some (minimum measure of) workers each period.⁵ Workers and firms discount the future with common factor $\beta \in (0, 1]$.

⁵This can be motivated by an assumption that firms cannot produce after a period of zero production.

For an employed worker, there is a “staying” probability of $\delta \in (0, 1)$, each period, with workers exogenously separating with probability $1 - \delta$; separated workers must seek work at a different firm; of the separated, a proportion ϕ remain in the same sector while $(1 - \phi)$ are distributed evenly across sectors.⁶ Likewise $(1 - \phi)$ of the unemployed move to other sectors. (Thus all movements between firms and between sectors are due to exogenous separations and workers cannot choose, for example, which sector to move to.) Separation occurs at the end of a period so that separated workers who find a job in the following period do not suffer any unemployment. We assume there are a large number of workers relative to the number of firms, and we normalize the ratio of workers to firms to be one in each sector and each period.⁷ We assume that the “spot wage”/full employment solution ($N = 1$) is always greater than the unemployment consumption level, i.e., that $\partial F^m / \partial N(1, s_t) > \underline{c}$ all t .

The shock s_t follows a stochastic process taking a finite number of possible values, and with initial value s_1 , which we specify in more detail below. Let $h_t \equiv (s_1, s_2, \dots, s_t)$ be the history at t . The labour market offers a worker currently looking for work in sector m (at the start of t , discounted to t) of $\chi_t^m = \chi_t^m(h_t)$: this is an average of the utility from remaining unemployed and that from getting a job where the weight depends on the probability of being hired as described below. A firm must offer at least $\chi_t^m(h_t)$ to prevent its workers from quitting, and this is also the minimum utility that must be offered to hire: We assume that the firm can hire any number of workers by offering at least χ_t^m (and cannot hire otherwise). So the labour market is modelled as being competitive.

We summarise the timing as follows. At date 1 each firm in sector m offers a

⁶We want to include multiple sectors in the model for the later simulation exercise. The fact that generally some sectors will have positive unemployment while others have zero unemployment means that the model does not have an aggregate unemployment rate that is often at a minimum bound.

⁷We keep the labour force fixed, although extending the model to allow exit and entry is straightforward and does not affect the results.

single state-contingent wage contract $(w_t^m(h_t))_{t=1}^T$ to which it is committed. Workers then can accept contracts and period 1 production takes place. At the end of period 1, a firm loses a fraction $(1 - \delta)$ of its workforce due to exogenous separation, as described above. At the start of each subsequent period $t \geq 2$, firms and workers observe s_t . Workers may quit costlessly at this point and join the pool of those previously separated, the unemployed and new entrants to the sector, facing the same probability of employment (so receive $\chi_t^m(h_t)$), but may not switch sectors. However, provided the continuation utility offered by the contract at least matches χ_t^m , the firm is able to retain its staff and hire in as many new workers as it requires from the pool of those looking for work. Production takes place and wages $w_t^m(h_t)$ are paid, and so on.

3.1 The Firm's problem

We work with a representative firm in sector m . At the start of date 1, after s_1 is observed, as just stated, firms in each sector m commit to contracts $(w_t^m(h_t))_{t=1}^T$, $w_t^m(h_t) \geq 0$, which we assume are not binding on workers. *We assume equal treatment within the firm:* a worker joining subsequently, at τ after history h_τ , is offered a continuation of this same contract: $(w_t^m(h_t))_{t=\tau}^T$. (This is to be contrasted with the case where discrimination is permitted where a worker joining at τ would be offered a contract which in principle may be unrelated to that offered to previous cohorts.) To avoid cluttering the notation, we omit sector superscripts in what follows unless necessary (so an omitted superscript implies that the sector is m). Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract, defined recursively by:

$$\begin{aligned} V_t(h_t) = & u(w_t(h_t)) + \beta[E\delta V_{t+1}(h_{t+1}) + \\ & (1 - \delta)\phi\chi_{t+1} + \sum_{m' \neq m} (M - 1)^{-1} (1 - \delta)(1 - \phi)\chi_{t+1}^{m'} | h_t], \end{aligned} \tag{1}$$

with $V_{T+1} = 0$, where E denotes expectation, and the terms involving χ_{t+1} and $\chi_{t+1}^{m'}$ reflect the utility after exogenous separation, if the worker respectively remains in the same sector or moves to another sector. Each firm also has a planned employment path $(N_t(h_t))_{t=1}^T$, where $N_t(h_t) \geq 0$.

Note that in (1) it is assumed that there are no layoffs, only exogenously determined separations. Our aim is to construct an equilibrium in which layoffs do not occur, largely because it substantially simplifies the analytics of the solution. Provided exogenous turnover is high enough relative to negative shocks, it can be shown that the firm would always want to hire in new workers, and would not want to replace any existing ones, so we henceforth assume the parameters are consistent with this.⁸

The problem faced by the firm, which takes the stochastic sequence of outside option values in its sector, $(\chi_t)_{t=1}^T$, as parametric (as well as those in other sectors), is:

$$\max_{(w_t(h_t))_{t=1}^T, (N_t(h_t))_{t=1}^T} E \left[\sum_{t=1}^T (\beta)^{t-1} (F^m(N_t(h_t), s_t) - N_t(h_t)w_t(h_t)) \right] \quad (\text{Problem DWR})$$

subject to

$$V_t(h_t) \geq \chi_t(h_t) \quad (2)$$

for all positive probability $h_t, T \geq t \geq 1$, and

$$w_t(h_{t-1}, s) \geq b(h_{t-1}, s) w(h_{t-1}), \quad (3)$$

for all positive probability h_{t-1} , all $s \in S$ with $\pi_{s_{t-1}s} > 0, T \geq t \geq 2$. (2) is the *participation constraint* that says that at any point the contract must offer at least

⁸For details see the arguments in Snell and Thomas (2009) which apply in the current setting *mutatis mutandis*. Note that given that the rate of separation is exogenous, it is changes in hiring that drive movements in unemployment in our model. This is consistent with Hall (2005), who argues that the separation rate in the US labor market is roughly constant (see also Pissarides (1986), Shimer (2005)), and that although job losses rise during recessions, the increase is usually very small in relation to the normal levels of separations. However, these conclusions have been disputed (see Elsby, Michaels, and Solon (2009)).

what a worker can get by quitting, and (3) is the *ad hoc* downward constraint that imposes that wages cannot fall at a rate faster than an amount which may depend on the current state (e.g., to capture inflation), given by $b(h_t)$. For $b = 1$ we have downward real wage rigidity, and for $b = 0$ we have the problem in which there is no downward constraint on wages. Downward nominal rigidity would be captured by $b(h_t) = p_{t-1}/p_t$, where p_t is the price level at t (prices play no other role in this model).

3.1.1 Equilibrium

We shall use a $*$ superscript to denote equilibrium values.⁹ We are looking for symmetric solutions, i.e., where all firms in a sector choose the same contract (though we do not need symmetry across sectors). To close the model we impose an equation specifying the equilibrium determination, given $(w_t^*(h_t))_{t=1}^T, (N_t^*(h_t))_{t=1}^T$, of the outside option in sector m :

$$\chi_t = \frac{N_t^* - \delta N_{t-1}^*}{1 - \delta N_{t-1}^*} V_t^* + \frac{1 - N_t^*}{1 - \delta N_{t-1}^*} U_t, \quad (4)$$

$N_0^* = 0$, where V_t^* is the equilibrium contract offer at t and U_t is the discounted utility of a worker who is unemployed at t —that is, who currently has failed to find a job. The number of workers who remain in a job from $t - 1$ is δN_{t-1}^* , the survival rate times the number employed in the sector at time $t - 1$. Thus the denominator of the coefficient on V_t^* is the number of workers not retained after $t - 1$, in other words the number seeking work at t , while the numerator is the number of hires at t . U_t in turn is given by

$$U_t(h_t) = u(\underline{c}) + \beta(\phi E[\chi_{t+1} | h_t] + \sum_{m' \neq m} (M - 1)^{-1} (1 - \phi) E[\chi_{t+1}^{m'} | h_t]), \quad (5)$$

⁹This subsection broadly follows Snell and Thomas (2009), *mutatis mutandis*, including proofs of the lemmas, which we therefore omit.

i.e., the utility from the reservation wage plus future utility from not having a job at the beginning of $t + 1$. Given the endpoint condition $\chi_{T+1}^{m'} = 0$, all m' , (4), (1) and (5) uniquely determine U_t , V_t^* and χ_t .

Note that there are two cases: if the labour market in sector m at time t clears, $N_t^*(h_t) = 1$, then from (4) $\chi_t(h_t)$ must offer the utility offered by other firms. In symmetric equilibrium, other firms in the same sector are offering an identical contract, and so it is the utility associated with this, $V_t^*(h_t)$, which must be offered. If, on the other hand, there is excess supply of labour,¹⁰ $N_t^*(h_t) < 1$, the outside opportunity will depend on the probability of getting a job.

We can summarise:

Definition 1 $\left((w_t^{*m}(h_t))_{t=1}^T, (N_t^{*m}(h_t))_{t=1}^T \right)_{m=1}^M$ constitutes a symmetric equilibrium if it solves Problem DWR for each m where $(\chi_t^m)_{t=1}^T$ is determined recursively from (1), (4) and (5).

Employment is determined by a standard marginal productivity equation (again suppressing sector superscripts):

Lemma 1 In a symmetric equilibrium $N_t^*(h_t)$ satisfies

$$\partial F(N_t^*(h_t), s_t) / \partial N = w_t^*(h_t). \quad (6)$$

This is a very useful implication of the combination of equal treatment and positive hiring. Effectively the firm can ‘neutralise’ an extra hire today by hiring δ fewer workers next period (possible by positive hiring), so that employment from next period on is unchanged. Notice that this requires equal treatment—if hires

¹⁰Intuitively, the case of excess demand for labour cannot arise in equilibrium, as an infinitesimally small increase in the wage would cure the individual firm’s supply problem. In contrast, because of equal treatment the case of excess supply can arise since workers cannot undercut.

next period were brought in on a different contract this neutralisation would not be possible. The only consideration is thus whether the extra worker makes a profit today, that is whether the current marginal product exceeds the wage; if it does the firm should hire more workers, so at an optimum there must be equality (because of positive hiring today the same logic applies in reverse if the current marginal product is less than the wage). In this argument the wage contract is held fixed at its optimum.

Suppose that at some t , the participation constraint binds. Then there must be full employment and the wage is determined by marginal productivity at full employment:

Lemma 2 *In a symmetric equilibrium the participation constraint binds at h_t if and only if $N_t^*(h_t) = 1$; moreover if the constraint binds then $w_t^*(h_t) = \partial F(1, s_t)/\partial N$.*

The argument for the first assertion of the lemma is simply that in a symmetric equilibrium if there is full employment, the quitting will give a worker the same utility as she can move immediately to another firm with an identical contract (by symmetry), so the worker is indifferent about leaving, i.e., the participation constraint binds. If, on the other hand, there is unemployment, then again by symmetry a worker must be worse off should she quit since at best she will get the same contract, but now there is a chance she will end up unemployed. Consequently the participation constraint must be slack.

We define $\underline{w}_s^{*m} = \partial F^m(1, s)/\partial N$, which in view of the second assertion of Lemma 2 is the equilibrium wage when the participation constraint binds in state s , but note that it would also be the wage in a spot version of the model. (The assertion follows from noting that full employment implies, from Lemma 1, that the wage must be at the spot level.)

The above is very useful as Lemma 1 tells us that if the contract wage is below the spot wage for that state, we get employment above unity, which is infeasible. So this case cannot occur¹¹ and the contract wage must always be at or above the spot wage. If wages are above the spot wage, on the other hand, there is unemployment, and so by Lemma 2 the participation constraint cannot bind. We stress that the foregoing does not depend on the form of the optimal wage contract (i.e., on optimally smoothing wages, taking into account downward rigidity constraints, etc.), but only on equal treatment, and an optimal hiring policy for a given contract, together with the hypothesis that firms always hire and that the equilibrium is symmetric across firms in the sector.¹²

To proceed to an explicit solution, and in order to facilitate the empirical analysis, we put more structure on the problem.¹³ This will allow us to assert, under certain conditions, that the wage updating rule in any sector m is of the following simple form: given w_t^* compute w_{t+1} under the hypothesis that the participation constraint at $t + 1$ is not binding; if $w_{t+1} > \underline{w}_{s_{t+1}}^*$ then the hypothesis is confirmed and w_{t+1} is the equilibrium wage; otherwise the constraint is binding and the equilibrium wage will be at $\underline{w}_{s_{t+1}}^*$. The structure will also allow us to demonstrate sufficient conditions for the symmetric hiring equilibrium to exist.

From henceforth assume each firm has technology given by, at time t ,

$$F^m(N, s_t) = M_t^{(m)} + a_t^{(m)} N^{1-\alpha} / (1 - \alpha), \quad (7)$$

¹¹Intuitively, the reason that excess demand for labour at time t is impossible is the following. A firm could, by an infinitesimal increase in its wage, attract as many workers as it wishes. Even with downward constraints, this will add at most a tiny amount to current and future wage costs, but it allows for a non-negligible increase in current profits.

¹²Another useful implication of this is that our assumption that the spot wage is always above the value of not working guarantees that equilibrium wages also always exceed this value. Thus in our later characterization, we shall not need to worry about running into the constraint that workers would rather not work than get or keep an employment contract.

¹³We also need the problem faced by the firm to be concave; concave production and utility functions are not sufficient to guarantee this.

for $\alpha \neq 1$, with $M_t \geq 0$ and for $\alpha = 1$, we specify $F^m(N, s_t) = a_t^{(m)} \log(N)$. $(M_t^{(m)}, a_t^{(m)})$ is a sector specific shock that depends on the aggregate shock s_t . Note that for $\alpha > 1$, F^m has an upper bound given by $M_t^{(m)}$, which given that we are modelling short-run production functions at the establishment or plant level, may be appropriate. For $\alpha < 1$, F^m has the more standard power function representation (corresponding to a short-run Cobb-Douglas production function). We assume that productivity shocks are not too bad (again dropping sector superscripts):

$$a_{t+1}/a_t > \delta^\alpha \quad (8)$$

with probability one (for example, with a log production function, this requires only that productivity does not fall at a rate equal to turnover; since the latter is typically estimated in the region of at least 30% on an annual basis, this is a mild restriction). This will ensure in the solution derived below, provided wages are sufficiently downward rigid, that firms will always need to hire in new workers (firms always loose more workers through exogenous turnover than they wish to), thus justifying our assumption that hiring always happens. We also assume henceforth that workers have per-period utility functions of the constant relative risk aversion family with coefficient $\gamma > 1$ described by $u(c) = c^{1-\gamma}/(1-\gamma)$.¹⁴ Assume $\alpha\gamma > 1$.¹⁵

We also have that the marginal product of labour equals $a_t N_t^{-\alpha}$, so that using (6),

$$N_t = a_t^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}}. \quad (9)$$

Substituting $N_t = 1$ we find that the spot wage is $\underline{w}_t^* = a_t$.

¹⁴For $\gamma = 1$, set $u(c) = \log(c)$; all results go through.

¹⁵This is needed to make the optimisation problem concave. Hence if $\alpha < 1$ (Cobb-Douglas) we need the risk version coefficient to be greater than unity.

3.1.2 No uncertainty

First we deal with the case of no uncertainty (so that all sectoral productivity sequences are known at date 1 before contracts are entered into). In this case, we show that the wage will always be kept as low as possible subject to it never falling below the spot wage. With downward real rigidity this would imply that the equilibrium satisfies $w(h_t) = \max_{t' \leq t} \underline{w}_{s_{t'}}^*$.

As a first step towards proving this, we demonstrate a frontloading result. The lemma establishes a very useful fact *if the model is deterministic* (transition probabilities are zero or one): provided the downward constraint is not too weak (for example, with nominal rigidity, provided inflation is not too large), then wages will fall between any two dates by the maximum allowed by the downward constraint unless the participation constraint at the later date binds.

The intuition is as follows: if wages next period are not up against the downward constraint, then frontloading them by cutting next period's wage a small amount and simultaneously increasing the current wage to compensate workers does not violate any downward constraints. If in addition next period's participation constraint is not binding then these too will be satisfied at all dates. This will increase profits however. The reason is that because there is turnover—a number of the current workforce will be separated before next period—to compensate workers the current wage does not have to be increased too much as they discount the future wage by the probability of separation (in addition to the discount factor). The firm, however, puts greater weight on wages next period because it will have to pay them to replacement workers (new hires) as well as to the surviving incumbents. Thus the cut in future wages is valued more highly by the firm and profits rise. The argument works so long as (a) wages are not falling too quickly, as then risk-averse workers will need substantial additional compensation now for the steeper wage path, and

(b) firms are hiring in new workers. Thus the downward constraint must not have $b(h_t)$ too small, and also negative productivity shocks should not be so severe that firms do not want to hire in new workers in some periods, a condition we had already assumed in order to solve the model.

Lemma 3 *Suppose there is no uncertainty. Then there exists a $\underline{b} < 1$, such that if $b(h_{t+1}) > \underline{b}$, all t , the following must hold: If in a solution to Problem DWR for sector m the downward rigidity constraint does not bind in sector m between t and $t + 1$ then the participation constraint binds at $t + 1$.*

Proof. We use time subscripts rather than history dependent functions as there is no uncertainty (and suppress sector m superscripts). Suppose to the contrary of the claim $w_t > b_t w_{t-1}$ but the participation constraint does not bind at $t + 1$. Starting from the optimal contract, consider reshuffling wages between t and $t + 1$ as follows: decrease the wage at $t + 1$ by a small amount $\Delta w_{t+1} > 0$ so that worker utility falls by $\Delta > 0$, and increase the wage at t by Δw_t so that utility rises by x , and so as to leave the worker indifferent; do not change the contract otherwise. This implies that

$$-\delta\beta\Delta + x = 0 \tag{10}$$

(where $u'(w_{t+1})\Delta w_{t+1} \simeq \Delta$ and $u'(w_t)\Delta w_t \simeq x$). This frontloading satisfies all participation constraints: worker utility falls at $t + 1$ but the constraint was initially slack by hypothesis, and so from this point on constraints are satisfied; similarly, participation constraints are also satisfied both at t and earlier because utility is held constant over the two periods. A sufficiently small change also satisfies the downward rigidity constraint because at $t + 1$ it was slack, while at t , $w_t + \Delta w_t > w_t \geq b_t w_{t-1}$, and at $t + 2$, $w_{t+1} - \Delta w_{t+1} < w_{t+1} \leq w_{t+2}/b_{t+2}$. We write $\Pi(u_t; a_t)$ as the static profit function at productivity level a_t (we can suppress M_t which only shifts profits

up or down; see (15) below for the explicit function) when workers receive a current-period utility of $u_t (= u(w_t))$, and $N(u_t; a_t)$ for the corresponding optimal labour demand. The optimal contract must generate profits of $\Pi(u_t; a_t)$ at t (the choice of N_t does not affect the other constraints, so N_t must be chosen to maximise current profits at the contract wage). The change in profits (viewed from h_t) arising from the frontloading is

$$\Delta P \simeq \beta \Pi'(u_{t+1}; a_{t+1}) \Delta - \Pi'(u_t; a_t) x. \quad (11)$$

Define $\varepsilon := \min_{2 \leq t \leq T} [\delta^{-\alpha} a_{t+1}/a_t - 1]$, where, by (8), $\varepsilon > 0$, so that

$$\delta^{-\alpha} a_{t+1}/a_t \geq 1 + \varepsilon. \quad (12)$$

From Hotelling's Lemma (converting wages to utilities), $\Pi'(u; a) = -N(u; a)/u'(w)$.

Thus,

$$\frac{\Pi'(u_t; a_{t+1})}{\Pi'(u_t; a_t)} = \frac{N(u_t; a_{t+1})}{N(u_t; a_t)} = \frac{a_{t+1}^{\frac{1}{\alpha}}}{a_t^{\frac{1}{\alpha}}} \geq \delta (1 + \varepsilon)^{1/\alpha}, \quad (13)$$

where the second equality follows from optimal labour demand $N = a^{\frac{1}{\alpha}} w^{-\frac{1}{\alpha}}$ (given that u_t and hence wages are constant in the ratio), and the inequality follows from (12).

Next, $\Pi(\cdot; a)$ is a concave function: Consider the static problem of maximizing profits given that workers receive utility u , so that $w = ((1 - \gamma)u)^{1/(1-\gamma)}$. Substituting from the condition that the marginal product of labour equals the wage:

$$N = a^{\frac{1}{\alpha}} w^{-\frac{1}{\alpha}} \quad (14)$$

yields profits of

$$\Pi(u; a_t) \equiv M_t + \frac{a_t^{\frac{1}{\alpha}} \alpha ((1 - \gamma)u)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1 - \alpha}. \quad (15)$$

As $\alpha\gamma > 1$, this is a strictly concave function of u .

Given that wages rise at a gross rate greater than b , then if $\gamma > 1$ (so utilities are negative; a similar argument, though with some inequalities reversed, applies for

$\gamma < 1$ and we omit it),

$$\frac{u_{t+1}}{u_t} = \left(\frac{w_{t+1}}{w_t} \right)^{1-\gamma} < b^{1-\gamma}. \quad (16)$$

Then from (15),

$$\frac{\Pi'(u_{t+1}; a_{t+1})}{\Pi'(u_t; a_{t+1})} = \left(\frac{u_{t+1}}{u_t} \right)^{\left(\frac{\alpha\gamma-1}{\alpha(1-\gamma)} \right)} > b^{\left(\frac{\alpha\gamma-1}{\alpha} \right)}, \quad (17)$$

where the inequality follows from (16). Substituting (10) into (11) yields

$$\begin{aligned} \Delta P &\simeq \beta \Pi'(u_{t+1}; a_{t+1}) \Delta - \Pi'(u_t; a_t) x \\ &= \beta \Delta \Pi'(u_t; a_t) \left(\frac{\Pi'(u_{t+1}; a_{t+1})}{\Pi'(u_t; a_t)} - \delta \right) \\ &> \beta \Delta \Pi'(u_t; a_t) \left(\delta (1 + \varepsilon)^{1/\alpha} b^{\left(\frac{\alpha\gamma-1}{\alpha} \right)} - \delta \right) \end{aligned}$$

where the inequality follows from (13) and (17). Thus, provided $b \geq (1 + \varepsilon)^{-\frac{1}{\alpha\gamma-1}}$, $\Delta P > 0$. As the initial contract was assumed optimal, this is a contradiction. Given $\alpha\gamma > 1$, $(1 + \varepsilon)^{-\frac{1}{\alpha\gamma-1}} < 1$. Hence setting $\underline{b} = (1 + \varepsilon)^{-\frac{1}{\alpha\gamma-1}}$, the assertion of the lemma follows. ■

We can now show that wages rising by the minimum given by the downward constraint, unless this takes wages below the spot wage in which case the wage is set to the latter, constitutes an equilibrium. We use a * to denote equilibrium values.

Proposition 1 *Suppose there is no uncertainty and that $b(h_t) > \underline{b}$, all t . Then there is a symmetric equilibrium in which in each sector m , $w_{t+1}^{*m} = \max\{b(h_{t+1}) w_t^{*m}, a_{t+1}^m\}$, $t \geq 2$ and $w_1^{*m} = a_1^m$.*

Proof. Suppose all other firms follow the putative equilibrium strategy and hire so that they are on their labour demand curves, i.e., marginal product of labour equal to the wage (this defines $(\chi_t)_{t=1}^T$ from (4)) and consider the optimal strategy of a potential deviant firm (which exists by standard arguments) in sector m . We show that any optimal strategy must coincide with the putative equilibrium strategy.

Again we drop sector superscripts and write b_t for $b(h_t)$. (i) If at T , the final wage in the deviant strategy $w_T < w_T^*$, then if $w_T^* = a_T$, there is full employment and so to satisfy the participation constraint a wage $w_T \geq w_T^*$ must be paid (see the remark below Lemma 2), so the participation constraint would be violated by the deviation strategy and it would be infeasible; on the other hand, if $w_T^* < a_T$, then at $t - 1$, $w_{T-1}^* = w_T^*/b_T$ by definition of the equilibrium strategy, and $w_T < w_T^*$ plus downward rigidity implies $w_{T-1} \leq w_T/b_T < w_T^*/b_T$, so the deviation contract offers less discounted utility at $T - 1$. Again, if the participation constraint binds at $T - 1$ for the equilibrium contract, the participation constraint would be violated for the deviant, and if it does not bind, we can extend the argument back to $T - 2$, etc. As soon as the participation constraint binds for the putative equilibrium contract (it must bind at $t = 1$ by $w_1^* = a_1$, as this implies full employment in the sector and hence by (4) a binding constraint), we will get a contradiction. (ii) If at T , $w_T > w_T^*$, then the participation constraint is slack for the deviant contract and so by Lemma 3 (recall we are looking at an optimal deviation contract, i.e., a solution to Problem DWR), $w_{T-1} = w_T/b_T$ and as $w_T^* \geq b_T w_{T-1}^*$, $w_{T-1} > w_{T-1}^*$. If at $T - 1$ the participation constraint binds for the deviant contract, it would be violated for the equilibrium contract, which is impossible. Thus it cannot bind at $T - 1$, and we can work backwards to the point where it last binds for the deviant (it must bind at least at $t = 1$ as otherwise cutting w_1 would improve profits without violating any constraint), at which point again the deviant strategy offers higher discounted utility, a contradiction. We conclude that $w_T = w_T^*$. By similar arguments we can show $w_{T-1} = w_{T-1}^*$, and work backward to establish equality of the two contracts. Thus deviation is not profitable. ■

3.2 Discussion

Thus, under the conditions of the proposition, the equilibrium wage contract can be computed in a simple recursive fashion, starting at the spot wage in period 1, and then proceeding by reducing wages if possible, due to the frontloading incentive, to the extent permitted by the downward constraint and the need to stay above the spot wage (i.e., to satisfy the participation constraint). The intuition behind the latter is as follows. If the equilibrium wage were to fall below the spot wage, given that each firm is on its labour demand function, there would be excess demand for labour. Each firm would have an incentive then to slightly raise its wage above that of its competitors in order to attract as many workers as it wants, so this could not be an equilibrium. The equilibrium must occur when all wages are raised to exactly the spot wage, for then a firm would not want to pay less (its workers would leave as there is full employment), and nor would it want to pay more (because of frontloading it will, in the putative equilibrium, be up against the downward constraint in future unemployment states, so that paying more now would force it to pay more in the future, raising wage costs for no benefit).

To illustrate the solution, we take the actual productivity series (TFP) over the period 1955 to 2001 for one of the manufacturing sectors that we use in the empirical exercise (see Section 4 for details). Figure 1 displays productivity and simulated wages. The spot wage equals the productivity level (the thicker line), and Proposition 1 says that wages are always at least at this level, but otherwise fall at the rate given by the downward constraint. Whenever the wage lies above productivity, the labour market fails to clear (and the participation constraint does not bind); the larger the percentage gap, the larger is unemployment (as $N_t = (w_t/a_t)^{-\frac{1}{\alpha}}$). We show three wage simulations. For much of the time they are coincident with the spot wage. The horizontal broken lines represent full downward rigidity ($b(h_t) = 1$).

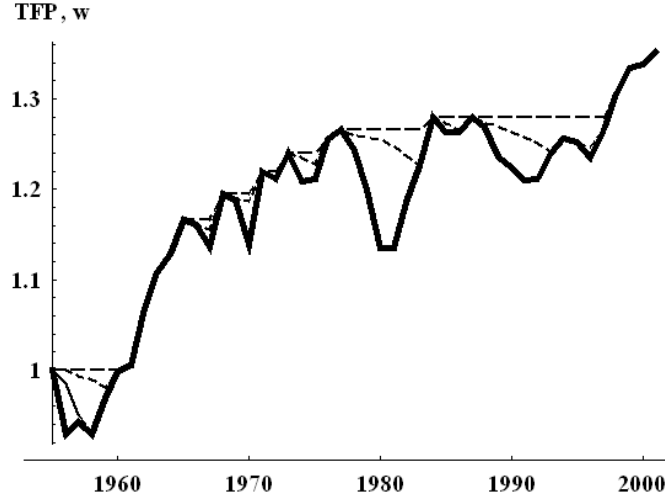


Figure 1: TFP (thick line) and wage simulations

The shorter broken line gives the path predicted in Snell and Thomas (2009), and is discussed below. The thin continuous line only visible before 1960 represents downward nominal rigidity (using the CPI to represent prices). The reason the latter is mostly coincident with productivity is that over the period 1955-2001 inflation in most years is greater in absolute value than negative sectoral productivity growth, so that the nominal rigidity constraint has little bite. We return to this issue below in Section 4.

We again stress that it is the interplay of the two constraints which matters for unemployment outcomes. Downward rigidity in the absence of equal treatment, would, in this model, not lead to deviations from full employment. In fact, individual wages would be the same as those predicted by Beaudry and DiNardo (1991), where wages are downward rigid but rise to prevent workers from quitting when wage offers from other firms rise above what was previously offered to a worker.¹⁶ Critically, this downward rigidity only applies to ongoing contracts, and would not apply to

¹⁶Under the productivity process they assume, this happens when productivity is higher than previously attained during a worker's current tenure. This is the equilibrium outcome in the absence of both constraints, given risk averse workers, so the ad hoc downward constraint would play no role.

the wage contracts offered to new hires, and consequently the labour market would always clear. Alternatively, equal treatment *per se* does not lead to deviations from Walrasian outcomes. If workers were risk neutral, for example, then wages which tracked the spot wages would be optimal—firms would be able to hire workers for the minimum possible discounted wages but would still satisfy the participation constraints.

When the two constraints coexist, however, any wage rigidity for incumbents is also transmitted to new hires. And we saw in Lemmata 1 and 2 how in this model equal treatment leads to the convenient property that wages above spot wages imply unemployment and a non-binding participation constraint. Combined with the other implication of Lemma 2 that wages must be at least at the spot level, we can see that positive productivity shocks—which push spot wages and hence contract wages up—followed by negative ones will lead to wages that are too high to clear the market.

This property that wages are at least equal to spot levels, and sufficiently positive shocks therefore push wages up (so that they equal the spot wage and full employment ensues), is a convenient implication of equal treatment and perfectly competitive labour markets/perfectly mobile labour in this framework. It answers, at least in this simple model, a question posed at the beginning of the paper: given that adverse shocks which can leave wages too high should be anticipated by forward looking firms, what determines the actual level of wages before the adverse shock hits? A striking illustration of this point arises if we consider a two-period model with high productivity \bar{a} in period 1 and low productivity \underline{a} in period 2, and suppose there is full downward rigidity ($b(h_2) = 1$). Proposition 1 asserts that the solution is $w_1 = w_2 = \bar{a}$, so that the wage in period 1 equals the high period 1 spot wage, and in period 2 the wage is too high to clear the market, even though all firms anticipate

this occurring.¹⁷

However, we conjecture that if labour was not perfectly mobile, the conclusion would change. In a high productivity state a firm would face a trade-off between the benefits of raising its wage to retain/attract workers and the future costs of carrying a higher wage forward which may constrain it in future adverse states. So one would expect downward rigidity to constrain upward movements too.

In Snell and Thomas (2009) rigidity arises due to the desire to smooth wage movements over time. If wages fall too fast then this imposes costs on the firm because it has to compensate workers for this variability. On the other hand, the same frontloading motive that was identified in the current model exists, so that firms when unconstrained will want wages to fall over time. Balancing these two forces provides a limit on how fast wages will fall. A problem, empirically, with appealing to risk aversion as the explanation of wage rigidity in that model is that to get plausible results it is necessary to assume that either workers are very risk averse (so it is very costly to have wage variability), that the rate of turnover is implausibly small (so the frontloading incentive is small) or that workers are more patient than firms (which again weakens the frontloading incentive).¹⁸ This suggests that the performance of the model may be improved if other sources of wage rigidity are present.

¹⁷Intuitively, this is an equilibrium because a deviant firm cannot offer a lower constant wage contract—there is full employment in period 1 so no worker would accept this when higher paying contracts are on offer. On the other hand, it cannot be the case that all firms offer a lower constant wage contract in equilibrium as then w_1 would be below \bar{a} and so there would be an excess demand for labour in period 1. A firm would then increase profits by deviating to a constant wage contract offering an infinitesimally higher wage, so that it could hire as many workers as it chooses. A higher constant wage contract cannot be an equilibrium as then w_1 would be above \bar{a} and there would be unemployment; a firm could cut its wage offering and still attract as many workers as it wants.

¹⁸In Snell and Thomas (2009) it is argued that extending the model to allow for experience dependent separation rates can help to resolve this problem.

The current paper demonstrates that it is possible to incorporate wage rigidity directly into the model. Given that we assumed that workers are risk averse, the same forces limiting downward movements of wages are present in the current model, so that if the downward rigidity constraint was sufficiently slack, it would be the desire to smooth wages that would be the binding constraint.¹⁹

Finally, imposing two ad hoc constraints might lead one to think that there will be large gains to be made if firms and workers can find a way to avoid these constraints. Here we argue that this is not so obvious. Importantly workers who are employed benefit from the interplay of the two constraints. Equilibrium wages are never below spot wages and sometimes above. If the firm can get around equal treatment, then it would benefit by being able to hire in new workers in depressed states at lower wages, but this does not benefit incumbent workers. Indeed, to the extent that the firm is unable to commit not to replace incumbents by cheaper new hires, this has the potential for making incumbents worse off given that it is in unemployment states that the firm would attempt to replace them.

On the other hand, if the downward rigidity constraint is relaxed, a firm can benefit (i.e., profitably deviate) from a policy that pays higher than spot wages in the beginning (and when the labour market clears) but—following the frontloading argument—lower wages than other firms when there is unemployment. This would allow it to satisfy participation constraints in good states but take advantage of cheap workers in bad states. Whether attempting to move to such a policy seems reasonable depends on one’s view of why downward rigidity might exist in the first place. There may be an element of implicit contracting involved; in fact it can be shown that if firms follow an implicit contract in which real wages are downward

¹⁹Some risk aversion is needed in order to make the profit function concave in the proof of Lemma 3. In Figure 1 the simulation with downward nominal rigidity lies below that from the Snell-Thomas model. This implies that under the parameters assumed in the latter, the nominal constraint would not bind.

rigid then each firm will have an incentive to keep to the equilibrium contract that we derived—it will be incentive compatible to raise the wage to the spot level when the incentive constraint binds even if the state is not verifiable. In such an informational environment, workers may be suspicious of a contract which allows firms to cut wages considerably, making the deviation contract mentioned above unpalatable. Bewley’s evidence seems to support this view of downward rigidity as being part of an implicit contract: “the main drawback of pay cuts is that they fill the air with disappointment and *an impression of breached promise* [emphasis added]” (Bewley (1998, p. 480)).²⁰

3.2.1 Uncertainty

The above arguments do not generalise to the case of uncertainty. Lemma 3 may fail as the frontloading of the wage contract between periods t and $t + 1$ in a particular state may now affect future wages in other possible states at $t + 1$ —the wage at t is increased, so if the downward constraint binds in some other state at $t + 1$ this will imply the wage increases in that state, which may be costly. However, we can show that if $T = 2$ this problem cannot arise. For simplicity assume $b(h_2) = 1$ all h_2 (downward real rigidity).

Proposition 2 *If $T = 2$, and $b(h_2) = 1$ all h_2 , there is a symmetric equilibrium in which $w_t^* = \max_{t' \leq t} \underline{w}_{t'}^*$.*

Proof. To establish this we convert the firm’s choice variable (contract) from wages $(w_t(h_t))_{t=1}^2$ to utilities $(u_t(h_t))_{t=1}^2$. We can formulate Problem DWR faced by the firm as:

$$\max_{(u_t(h_t))_{t=1}^2} E \left[\sum_{t=1}^2 (\beta)^{t-1} \Pi(u_t(h_t), a_t) \right] \quad (\text{Problem A}^R)$$

²⁰Fehr, Gächter, and Kirchsteiger (1997) provide experimental evidence that implicit contracts can be enforced by such reciprocity.

$$\text{subject to} \quad \tilde{V}_t(h_t) \geq \chi(h_t) \quad (18)$$

for all positive probability h_t , $2 \leq t \leq T$, and

$$u_t(h_{t-1}, s) \geq u(h_{t-1}), \quad (19)$$

where $\tilde{V}_t(h_t)$ is defined recursively as before by:

$$\tilde{V}_t(h_t) = u(w_t(h_t)) + \beta E[\delta V_{t+1}(h_{t+1}) + (1 - \delta)\chi_{t+1} | h_t], \quad (20)$$

with $\tilde{V}_3 = 0$. The maximand is strictly concave (see the proof of Lemma 3) and the constraints are linear. The Slater condition is satisfied by, for all h_t , $u_t(h_t) = u(w^*(h_t) + \varepsilon)$, for $\varepsilon > 0$. Moreover it is straightforward but tedious to show that the Kuhn-Tucker conditions are satisfied at the putative equilibrium contract, hence the putative solution solves Problem DWR. ■

For $T \geq 3$ we can construct counterexamples to the putative equilibrium, but only if there are shocks sufficiently bad that productivity falls in a range close to δ . We do not have an analytical result however.²¹

4 Simulations

In this section we assess to what extent the model is consistent with some relevant labour market stylised facts. In particular, we gauge whether the model can generate a plausible degree of unemployment volatility from measured total factor productivity shocks using US post war aggregate unemployment and productivity (TFP) data from the Bureau for Labor Statistics (BLS), and how well wage/unemployment regressions on simulated data correspond to existing stylised facts. In a single sector, unemployment falls to zero whenever the productivity shock is not too bad.

²¹We simulated models with iid two-state multiplicative productivity processes up to $T = 13$. For example, for $\delta = .8$, we confirmed the putative equilibrium for a wide range of other parameter values, provided a_{t+1}/a_t was at least approximately .82 with probability one.

Using the multisector model, however, in which each sector will be subject to an idiosyncratic productivity shock, we will obtain more realistic unemployment levels because it is less likely that all labour markets will simultaneously clear; moreover when the aggregate productivity shock is positive, there will be more sectors with low unemployment and consequently aggregate employment is likely to be lower.

Given knowledge of the model’s parameters, given an initial time period where there was full employment and given a TFP series, it is possible to generate the sectoral “real wage” series that would be predicted by versions of our theory. We can only assert that this is an equilibrium if there is no uncertainty, so each sectoral sequence is perfectly anticipated, although as noted above, we conjecture that this is also an equilibrium with uncertainty provided shocks are not too negative. It is then possible to derive the corresponding implications for unemployment (rates), and also the relationship between real wages and unemployment.

In accordance with the theory developed earlier, we generate for each sector separate predicted wage and unemployment series, using actual U.S. manufacturing industry multifactor productivity processes for the 17 manufacturing sectors provided by the BLS for the period 1949-2001.²² This fixes the variability of shocks and their correlation across sectors, and also allows us to generate a simulated unemployment series which can be directly compared to the data. None of our theoretical results depended on the sectors being of the same size, so our results readily extend to this asymmetric case; indeed allowing sector sizes to vary over time would also be a straightforward extension. We then aggregate the model’s predicted unemployment for each of these sectors using mid-period unemployment shares as weights²³ (we start simulations at full employment and spot wages in 1949, allowing 6 years for unemployment to develop in each sector, so we only use the period 1955-2001

²²This is the only sectoral TFP series available for such a long time scale and collected on a consistent basis.

²³We add 3.5% to represent a constant frictional rate.

for our results). Recall that this involves assuming that each sectoral labour market is segmented so we can compute unemployment in each sector independently, and then aggregate.

We do two things. First we look at the predicted unemployment series and compare this with the actual US experience. Secondly, we examine the relationship between real wages and unemployment over the business cycle.

To simulate unemployment, we consider in Figure 2 the cases of full downward real rigidity (thick broken line) and downward nominal rigidity (thin broken line); aside from the level of rigidity, there is only one degree of freedom, the choice of α —the curvature of the production function. For downward real rigidity we use a logarithmic production function ($\alpha = 1$).²⁴ However it should be remarked that as manufacturing is only a fraction of the entire economy (and becoming smaller over time), comparing the predicted unemployment rate for our model manufacturing economy with the general unemployment rate, as we do, is inappropriate. Nevertheless, the value of α essentially determines only the extent to which a non-market clearing wage translates into unemployment; it does not affect the equilibrium wage path but only magnifies the unemployment fluctuations as it falls in size. Thus if the remaining economy was composed of a residual sector with constant unemployment, the similar fluctuations in the economy wide unemployment rate would result from choosing a lower value for α (which enhances fluctuations). For the downward nominal rigidity we use a lower value of $\alpha = 0.2$ (the series is even flatter with $\alpha = 1$). Even so, there is far too little variation in unemployment, and as already remarked in Section 3.2, the constraint only has bite at the start of our period and towards

²⁴Estevão and Wilson (1998) found a short-run demand elasticity ranging between close to zero and -0.71 with aggregate data based on BLS manufacturing data for a similar period that we study, and of between -0.5 and -0.89 at the 4-digit industry level for manufacturing. An elasticity of -0.71 would correspond to a value of α of about 1.4. Hamermesh (1993) reports that an elasticity of around -0.3 is typical. Note that the value of the risk aversion parameter, γ , does not affect the solution (it is relevant for the value of b , however).

the end, when the inflation rate was low.

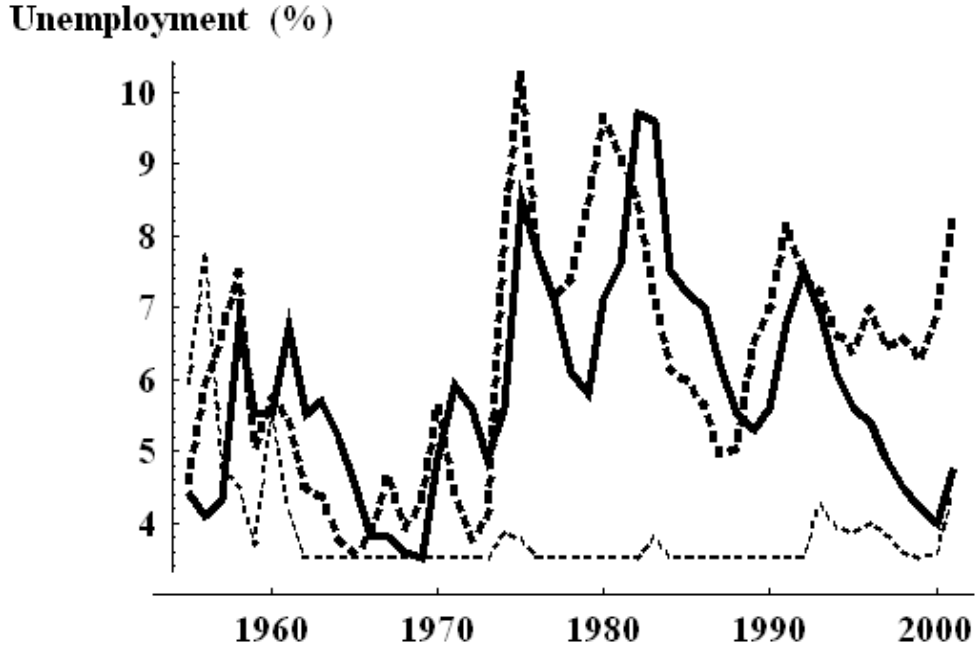


Figure 2: Unemployment simulations (broken lines) and actual annual US rate (solid line), 1955-2001

Given there is only one degree of freedom (together with an assumed constant frictional rate of unemployment which shifts the simulated series up or down), the simulation with downward real rigidity tracks unemployment surprisingly well until towards the end of the simulation. The actual standard deviation of the unemployment rate was 1.5% over our period; the simulated series has a standard deviation of 1.7%. What seems to be happening towards 2001 is that some sectors experience a sufficiently long trend of poor productivity shocks that unemployment in those sectors builds up with completely downward rigid wages. It is clearly unrealistic to suppose that labour wouldn't in the long-run move out of these sectors, so even if downward rigidity was appropriate we should expect to see lower unemployment towards the end of the period than the simulation suggests. Allowing wages to fall somewhat reduces simulated unemployment particularly towards the end of the simulation. However keeping to the logarithmic production function we find

that unemployment variability falls; for example if real wages can fall at 1% p.a. ($b(h_t) = 0.99$) we find the standard deviation falls to 1.0%, and for a fall of 2%, we get a standard deviation of 0.7%.²⁵

For our second exercise, we follow the studies of real wage cyclicalities that have looked at how wages respond to contemporaneous unemployment movements. While there is a huge literature on this, a very rough summary would be that wages are roughly acyclical, or mildly procyclical, with panel studies consistently pointing towards the latter. For example, using the PSID for men over the period of 1968-69 to 86-87, Solon, Barsky, and Parker (Feb 1994) found that a one percentage point reduction in the unemployment rate leads to a rise in the real wage rate of 1.4 percent. Similar estimates are found in Shin (Oct 1994) and Devereux (Jul 2001). From our simulations, we can regress real wage changes on changes in unemployment to replicate the typical regression undertaken in the panel studies.

Most studies which use longitudinal data of real wage cyclicalities, following Bils (1985), estimate the following:

$$\Delta \ln w_{it} = \beta \Delta U_t + \lambda t + \alpha' X_{it} + \varepsilon_{it}, \quad (21)$$

where $\Delta \ln w_{it}$ is the difference between the natural logarithm of worker i 's real wage rate in year t and his log real wage in year $t - 1$, ΔU_t is the year-to-year change in the unemployment rate, and X_{it} is a vector containing an intercept and time varying individual characteristics. The equation also includes a linear time trend (i.e., corresponding to a quadratic in t in levels). We ran the regression equation (21) on our 47 years of simulated data. Since all workers have identical productivity

²⁵Real wage falls in this region do not seem implausible. For example, Elsby (2009) charts the distribution of real wage changes in the PSID over 1983-1992, a relatively low inflation period (so surprise inflation is less likely to lead to unanticipated real wage falls); there is a spike around 2-4% for real wage falls, and they rarely exceed about 6%. Likewise Christophides and Stengos (2003) find from Canadian wage contract data in the unionized sector that most real wage reductions in the 1990s were of the order of 1-2%.

in our model, there are no distinguishing individual characteristics.

Under full downward real rigidity and a logarithmic production function, the estimate of β , the unemployment semi-elasticity of the wage, is -0.25 (all reported coefficients are significant). That it is negative is perhaps unsurprising—with full downward rigidity, for example, wages only change in a sector when there is full employment, in which case they rise. Aggregating across sectors, a fall in unemployment will tend to be associated with more sectors having full employment and consequently wages rising in more sectors. Again, allowing the real wage to fall by 1% and 2% we get respective β estimates of -0.44 and -0.72 .²⁶ While these are correctly signed, their magnitude is on the low side compared with the studies mentioned above. They do improve on the performance of the model in Snell and Thomas (2009) however. Running the same regression gives a coefficient of $+0.51$, incorrectly signed.

5 Concluding Comments

This paper has analysed a model with downward rigidities in which firms cannot pay discriminate based on year of entry to a firm. We solved for the dynamics of wages and unemployment under conditions of downward wage rigidity, where forward looking firms take into account these constraints. We found that the equilibrium could be solved for under conditions of certainty. Using actual productivity data based on the post-war US economy, we analysed the ability of the model to match certain stylised labour market facts, and found that it was able to generate sufficient variability of unemployment, and also match to an extent the empirical wage-unemployment relationship.

²⁶If we simultaneously change α to maintain the variability of unemployment, however, the coefficient stays close to the -0.25 estimate.

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